him to appoint delegates to Boston, he requested the Secretaries to send information to each of the Board of Officers and to Dr. R. E. Rogers and to Mr. Wm. A. Ingham that he nominated them as delegates to represent this Society and assist on the 26th of May at the Centennial Celebration of the American Academy of Arts and Sciences in Boston.

The meeting was then adjourned.

Astronomical Approximations. IV. Nodal Estimation of the Velocity of Light. By Pliny Earle Chase, LL.D., Professor of Philosophy in Haverford College.

(*Read before the American Philosophical Society, March* 19, 1880.)

The accuracy of my approximation to the apparent semi-diameter of the Sun* is confirmed by the following kinetic considerations, some of which, though seemingly of great fundamental importance and character, have been generally overlooked.

1. Matter has been usually regarded as composed of discrete particles. This hypothesis enters even into the kinetic theory of gases. If it is true, all force must be transmitted from particle to particle and time must, therefore, be required to overcome the inertia of masses.

2. Attraction and repulsion have been generally considered under the influence of central forces, varying inversely as the squares of the distances from the centres and, therefore, producing motions with variable velocity.

3. Waves, orbital undnlations, and other cyclical motions, are generally propagated with uniform or nearly uniform velocity, although they are often accompanied by subordinate movements with varying velocity. Variable velocities are often converted into uniform or nearly uniform velocities, as in the case of conical pendulums, planetary rotations and orbital revolutions.

4. In all undulations, and in all cyclical motions through an undulating medium, there are tendencies to synchronism. The synchronism may be complete, producing equal cyclical motions in equal cyclical times; or nodal, producing harmonic series of cyclical motions which are completed in equal times.

5. Newton showed that if a centripetal force varies as the distance of a body from the centre, all bodies, revolving in any planes whatsoever, will describe ellipses and complete their revolutions in equal times;† that bodies which move in right lines, running backwards and forwards alternately, will complete their several periods of going and returning in the

*Proceed. Am. Phil. Soc. xviii, 380.

† Principia, B. I, Prop. 47

same times ; * and that if a fluid be composed of particles mutually repelling each other, and if the density varies as the compression, the centrifugal forces of the particles will be reciprocally proportional to the distances of their centres,† as if indicating a reaction against a force which varies as the distance from the centre. The centripetal force is partially illustrated by the rotation of planetary bodies; the centrifugal, by the varying pressure of elastic atmospheres.

6. Such centripetal force as is above supposed, requires, for its complete manifestation, an omnipresence of activity, which is devoid of inertia, of ponderability, and of all other ordinary tests of material nature ; an activity which may, perhaps, properly be regarded as spiritual.

7. Laplace found himself obliged to recognize an activity in gravitation, which is propagated with at least 100,000,000 times the velocity of light.‡ This activity, he says, may be properly regarded as instantaneous. If it is spiritual, it may without difficulty be regarded as absolutely instantaneous throughout the universe. If it is material, it is difficult to conceive of any relation of elasticity to density which would be so great as 10,000,000,000,000,000 times that of the supposed luminiferous æther. Yet such are the requirements of the Newtonian law, that "the velocities of pulses propagated in an elastic fluid are in a ratio compounded of the subduplicate ratio of the elastic force directly, and the subduplicate ratio of the density inversely." § The importance of this law has been shown by the investigations of Graham, by the inquiries of English and German physicists into the relations between electromagnetic and luminous velocities, and by my own correlations of the force of solar rotation with the forces of light, gravitation and chemical attraction.

8. In any elastic or quasi-elastic medium, "if the distances be taken in harmonic [or arithmetical] progression, the densities of the medium at those distances will be in a geometrical progression." ‖ I use the term "quasi-elastic," in order to meet the views of Faraday, Preston ¶ and others, who prefer to treat all kinetic questions in accordance with lines of force.

9. The distances of projection, under uniform resistance, are proportioned to the living forces of projection, and inversely as the density of the resisting medium.

* Principia, B. I, Prop. 47.
† Ib. B. II, Prop. 23.
‡ Mec. Cel., X, vii, 22.
§ Principia. B. II, P. 48.
‖ Ib., B. II, P. 22.
¶ P. Mag., June, Sept., 1877. Preston does not give Maxwell's demonstration of the ratio $\sqrt{\frac{5}{9}}$, and neither he nor Maxwell seems to have been aware that I had used the same ratio of *vis viva* five years previously, in discussing results of gaseous energy (Proc. Am. Phil. Soc. Feb. 16, 1872, vol. xii, p. 394, foot-note). I showed that in the explosion of gases, the secondary centre of oscillation, on the return towards the centre, is at $\left(\frac{12}{18} - \frac{2}{3} \text{ of } \frac{12-9}{18} = \frac{5}{9}\right)$ of the extreme excursion.

10. In most, if not in all, physical investigations which introduce considerations of central force, velocity may be treated as a function of radius, time and mass.

$$v = F(r,\ t, m).$$

In undertaking to investigate successive conversions of spiritual, undulatory, and centripetal or centrifugal energy, astronomical phenomena furnish the most abundant, extensive, and varied opportunities for observation and illustration. Unfortunately, there is so much uncertainty in regard to the dimensions and densities of the principal heavenly bodies, that we can point to few results which are so precise as would be desirable. There are, however, some important indications of the operation of the foregoing laws, which are confirmed by terrestrial phenomena that are capable of very accurate measurement.

In Searle's Outlines of Astronomy, page 403, the following figures are quoted from the *Annalen der Sternwarte in Leiden*, on the authority of Kaiser. "The apparent diameter of Mercury, at a distance equal to the semi-axis major of Earth's orbit, ranges, according to different observers, from 5″.2 to 6″.9 ; that of Venus from 16″.6 to 17″.9 ; the apparent equatorial diameter of Mars from 9″.6 to 9″.2 ; its apparent polar diameter from 9″.4 to 9″.2. Similar disagreements appear in different estimates of the apparent diameters of Jupiter, Saturn, Uranus, and Neptune, at a distance for each planet equal to the semi-axis major of its orbit. Thus the equatorial diameter of Jupiter ranges from 39″.5 to 37″.1 ; its polar diameter from 37″.9 to 35″.1 ; the equatorial diameter of Saturn from 18″.5 to 16″.9 ; its polar diameter from 16″.8 to 15″.1 ; the diameter of Uranus from 2″.9 to 3″.6 ; that of Neptune from 2″.5 to 4″.4."

Kaiser adopts, as most probable values for the ratios of the several diameters to Earth's diameter, figures which I have increased by one per cent., in order to adapt them to more recent estimates of Sun's distance. These increased figures are given under r(I), in the following table. Under r(II) and in the subsequent columns, I give theoretical values which illustrate simple harmonic deductions from the above general function of velocity.

	Mass-denominator.	r(I).	r(II).	ρ.		g.	δ.
1. Mercury,	4865751	.34	.36	.387	$\rho^{\frac{2}{3}}$ =	.53	1.484
2. Venus,	427240	.97	.98	.723	$\rho^{\frac{2}{3}}$ =	.81	.822
3. Earth,	330985	1.00	1.00	1.000		1.00	1.000
4. Mars,	3093500	.53	.53	1.524	$\frac{1}{4}\rho$ =	.38	.719
5. Jupiter,	1047.88	11.04	11.02	5.203	$\frac{1}{2}\rho$ =	2.60	.236
6. Saturn,	3501.6	9.10	9.20	9.539	$\rho \div \rho_1$ =	1.12	.121
7. Uranus,	22600	3.60	3.60	19.184	$\rho \div t$ =	1.13	.314
8. Neptune,	19380	5.80	5.84	30.034		.50	.086
9. Sun,	1	109.11	109.05			27.83	.255

The value which I have assumed for Earth's mass is the one which I have deduced from velocities of rotation and revolution and Struve's constant of aberration. The other mass-denominators are the ones which are now generally adopted by astronomers. The semi-axes major are represented by ρ; the superficial equatorial acceleration of gravity, by g; the density, by δ; Earth's values being assumed as the units. In the expressions for g of Saturn and Uranus, $\rho_1 = 8.539$; $t = 16.9823$; ρ_1 being the mean distance between the centre of the belt of greatest condensation (Earth) and the primitive centre of rotary inertia (Saturn); t being the ratio between the time of Earth's rotation (a sidereal day) and the limiting time of satellite-revolution $\left(2\pi\sqrt{\frac{r}{g}}\right.$, at Earth's equatorial surface$\left.\right)$. It is impossible, at present, to assign any more probable values for the planetary radii than those which I have given under r(II). From those values and the masses, g and δ are readily found by the proportionalities, $\delta \propto m \div r^3$; $g \propto m \div r^2$. The relations of g to simple functions of the semi-axis major would be very striking, even if they were only approximately true. Although we cannot ascertain whether the relations are exact or not, the following considerations seem to increase the probability that they are correct indications of normal harmonic modifications of gravitating force by distance and time.

In comparing the gravitating force at the principal centres of early nebular activity, we may, then, acknowledge a strong probability:

1. That the force at the centre of reciprocity (Neptune), is one half as great as at the centre of the belt of greatest density (Earth).

2. That the force at the centre of rotary inertia (Saturn), is to the force at the centre of density, as the rotary centripetal force at Sun, is to the rotary centripetal force at Earth, the rotating tendencies being referred to Saturn as a centre.

3. That the force at the centre of nebulosity (Jupiter), is to the force at the centre of density, as the combined influence of the gravitating force at the centre of reciprocity and the rotary centripetal force at Jupiter, is to the combined influence of the corresponding forces at Earth.

4. That the force at the centre of nucleation (Sun), is equal to the velocity of light, divided by the time of a half-rotation; the half-rotation indicating the alternate oscillation of equatorial particles, from and towards the centre of gravity of Sun and Jupiter.

In extending the comparison to the subordinate centres within the belt of greatest condensation, we find a like probability;

5. That the force at Mars is to the force at the centre of density, as the combined force of gravitating *vis viva* at the centre of reciprocity and rotary centripetal force at Mars, is to the corresponding combined influences at Earth.

6. That the force at each of the inferior planets (Venus and Mercury), is represented by the ratio of projectile living forces to times of revolution.

Leaving the outer and somewhat doubtful territory, and entering upon surer ground, let us consider some of the obvious results of conversion of primitive force, with reference to centres of condensation.

1. If the velocity is constant, the radius of rotation or revolution must be proportioned to the time.

2. If radii, which were originally established by a constant velocity, are converted into radii of free revolution under equivalent central forces, the times of revolution must be proportioned to the $\frac{3}{2}$ power of the radii.

3. If radii of synchronous revolution and axial rotation are due to the action of a primitive constant wave-velocity, while nucleal radii are due to the collision of particles moving with parabolic velocity, the former velocity would be communicated in the time of a half-rotation, while the latter would be communicated in $\frac{\sqrt{2}}{\pi}$ of the same time.

4. Let us suppose that the ratio of the solar year to the terrestrial day originated in the constant velocity (1) of light, which is still represented by the equation of solar half rotation $\left(\frac{gt}{2} = v_\lambda\right)$; that the time-radii (r_t) were converted into radii of free revolution under equivalent central forces (2); and that corresponding nucleal radii (ρ_t) were established by parabolic collision (3). We should then have

$$\left(\frac{r_t}{\rho_t}\right)^{\frac{3}{2}} = \frac{\pi}{\sqrt{2}}$$

or, substituting the ratios which are represented by r_t and ρ_t,

$$\left(\frac{1 \text{ year}}{1 \text{ day}}\right)^{\frac{3}{2}} : \left(\frac{\text{Earth's semi-axis major}}{\text{Sun's semi-diameter}}\right)^{\frac{3}{2}} : : \pi : \sqrt{2},$$

The sidereal year is composed of the original nebular sidereal rotation and the 365.25636 additional sidereal rotations which are due to terrestrial condensation. Making these substitutions,

$$\left.\begin{aligned} &(365.25636)^{\frac{3}{2}} : x^{\frac{3}{2}} : : \pi : \sqrt{2} \\ &x = 214.5365 \end{aligned}\right\} \text{..................A.}$$

Among the obvious nodal influences of distance and velocity which may be reasonably supposed to have modified the kinetic undulations between the centres of density and of nucleation, the following may be specified :

1. The velocity of light, V_λ, or the projectile velocity which is equal to the sum of Sun's gravitating equatorial reactions during a half rotation, $\frac{gt}{2}$.

2. Sun's limiting velocity of revolution, $V_0 = \sqrt{gr}$, r being the equatorial radius.

3. Earth's limiting velocity of revolution, $v_0 = \sqrt{gr}$, at the equatorial surface.

4. Earth's superficial equatorial velocity of rotation, v_r.

5. Earth's semi axis major, ρ_0.

6. Earth's diameter, $2r$, or the major-axis of limiting synchronous linear, elliptical and circular oscillation.

7. Moon's semi-axis major, ρ_1.

8. The ratio of variation between Laplace's limit, r_1, and the nucleal radius, r_0; $r_1 \propto r_0^{\frac{4}{3}}$.

From Struve's constant of aberration, equation A, and other well-known measurements, we find: $V_\lambda = 214.5365 r_0 \div 497.827 = .4309458 r_0$; $V_0 = 214.5365^{\frac{3}{2}} \times 2\pi r_0 \div 31558150 = .00062564 r_0$; $v_0 = \left(\frac{3962.82 \times 32.088}{5280}\right)^{\frac{1}{2}} = 4.9075$; $v_r = 2\pi \times 3962.82 \div 86164 = .288974$; $V_\lambda \div V_0 = 688.815$; $v_0 \div v_r = 16.98237$. The combined nodal action of V_λ, V_0, v_0, v_r, ρ_0 and $2r$ are represented by the equation:

$$\left.\begin{array}{l} \dfrac{V_\lambda}{V_0} \div \dfrac{v_r}{v_0} = \dfrac{\rho_0}{2r} \\ 688.815 \times 16.98237 = \rho_0 \div 7925.64. \\ \rho_0 = 92{,}711{,}850 \text{ miles.} \\ V_\lambda = \rho_0 \div 497.827 = 186{,}233 \text{ miles} = 299{,}705 \text{ kilometers.} \\ r_0 = \rho_0 \div 214.5365 = 432{,}495 \text{ miles.} \\ \rho_0 \div r = 23{,}395.4. \\ \dfrac{\text{Sun's mass}}{\text{Earth's mass}} = 330{,}997. \end{array}\right\} \quad \ldots\ldots\ldots \text{B.}$$

If we adopt Newcomb's estimates of Sun's diameter, mass and distance (*Popular Astronomy*, p. 528), the nodal value of V_λ would be 185,334 miles; the value as determined by Struve's constant of aberration, 185,475 miles; the discrepancy being only $\frac{3}{40}$ of one per cent. Michelson's estimate (299,820 km.) is about $\frac{1}{26}$ of one per cent. greater than mine, and about $\frac{4}{9}$ of one per cent. greater than Newcomb's.

Newton's law of the ratio of elastic densities to distances (8), the law of projection under uniform resistance (9), and the ratio of variation between Laplace's limit and the nucleal radius, all are illustrated by the lunar equations:

$$\left.\begin{array}{l} \dfrac{v_0}{v_r} \times \dfrac{32}{9}\, r = \rho_1 = 60.3815 r. \\ \left(\dfrac{v_0}{v_r}\right)^{\frac{32}{9}} r = \rho_0 \textit{ very nearly} = 23{,}622 r. \end{array}\right\} \quad \ldots\ldots\ldots\ldots\ldots \text{C.}$$

J. J. von Littrow's estimate for Moon's semi-axis major, cited by Searle (p. 406), is 60.2778, or about $\frac{17}{100}$ of one per cent. less than the above result. The velocity of light, as deduced from the lunar value of ρ_0, is 188,040 miles, or nearly one per cent. greater than the value found in equations B.

I think no one will be likely to attach much weight to the larger value, but it is interesting on account of its indication of elliptical nebular influence, with a nucleal radius about one per cent. larger than Sun's present radius, and a major axis about one per cent. larger than Earth's present mean vector-radius. The nebular influence may be inferred from the fact that $\frac{32}{9} = 2 \times \left(\frac{4}{3}\right)^2$.